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ON GPS HEIGHTING IN LOCAL NETWORKS

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ABSTRACT

Determination of heights with help of GPS in local geodetic networks is still more actual respecting the fact that the GPS technology becames more and more effective with hardware progress, with improvements in measuring and evaluating procedures, and with better modelling of the disturbing influences. In comparison with GPS the employment of classical terrestrial measuring technologies is often more difficult namely in broken mountain environment.

In period 1998-2005 authors carried out repeated measurements of GPS baselines of various length and various height differences in local geodynamical network Sněžník and in other experimental areas. On ground of analyses of large GPS data sets the modified procedure for GPS observation was designed. The procedure is based on repetition of shorter static sessions separated by time intervals of optimal length. This technology represents an alternative to the ususal long static sessions, and is offering better effectivity of vertical GPS measurements with minimal loss of accuracy.

The paper presents detailed description of the modified procedure together with some statistical analyses of results. The possibilities of elimination or mitigation of some disturbing influences are discussed. Two testing vertical profiles were marked in Sněžník network– longitudinal profile in N-S direction, and transversal profile in E-W direction – which were measured in course of several years by classical method of very precise levelling, and also by modified GPS heighting procedure in repeated sessions. Results obtained contributed to the local quasigeoid model creation.

KEYWORDS: geodynamics, GPS, height measurements, geoid modelling

1. INTRODUCTION

From well known reasons the GPS derived vertical positional information is less accurate than the horizontal one. These reasons are satellite visibility (no visible satellites under horizon), satellite associated errors (orbits, clock bias), signal propagation errors (ionospheric/tropospheric delay), receiver associated errors (clock bias, antenna phase center offsets/variations), station associated errors (surroundings, centering, signal multipath and diffraction), secondary effects (solid earth tides, ocean and atmospheric loading etc.), and possible uncertainties in alignment to reference frames and to given vertical datum. In spite of that the GPS heighting becames more and more popular in present time and is considered as an alternative to classical terrestrial height measuring methods. GPS technology becames more and more effective with hardware progress, with improvements in measuring and evaluating procedures, and with better modelling of the disturbing influences. Employment of classical terrestrial vertical measuring technologies is often more difficult namely in broken mountain environment.

To achieve the accuracies desired, the relative GPS positioning methods with phase observations are used for the purposes of height determination. Observation modes include static, rapid static, kinematic, and pseudokinematic techniques. In last years the progress of real time technologies (RTK, or DGPS) can provide accuracies comparable with those of the classical techniques based on post-processing (Hofmann-Wellenhof et al., 2000). From methodological point of view the determination of heights using GPS is more complicated than by classical terrestrial methods. GPS is 3D positioning method which determines coordinates in global geocentric system (WGS-84). From practical reasons the spatial Cartesian coordinates X, Y, Z are transformed into geodetic coordinates B, L, h, and eventually into local topocentric coordinates n, e, u. Finally, the ellipsoidal heights are transformed to orthometric or normal heights. For this step the knowledge of (quasi)geoid undulations N_i is necessary, which are defined by

$$h = H + N \quad , \tag{1}$$

where h is ellipsoidal height, H is orthometric height and N is geoidal undulation.

The undulations can be determined using either absolute or relative methods. Geoidal surface is irregular and it is practically impossible to represent it by exact mathematical modeling function. Convenient is often an expression through spherical harmonics expansion. Satellite geoid can be modeled by a set of harmonic coefficients describing an Earth's gravity potential, see e.g. (Heiskanen and Moritz, 1987), (Teunissen and Kleusberg, 1998):

$$W_0 = \frac{G \cdot M_E}{r \cdot}\Big(1+\sum_{n=2}^{n'}\left(\frac{a_e}{r}\right)^n \sum_{m=0}^{n} P_{nm}(\sin\varphi)\cdot \\ \cdot\big(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\big)\Big) + \frac{\omega^2 r^2}{2}\cos^2\varphi \tag{2}$$

from which the geoid heights can be derived. P_{nm} are the associated Legendre's functions (normalized) of degree n and order m, C_{nm} and S_{nm} are the spherical harmonic coefficients, $G.M_E$ is the Earth's geocentric gravitational constant, and a_E is the equatorial radius of the Earth. In similar way the disturbing potential T can be expressed, and then the geoid undulation is computed from

$$N = \frac{T-(W_0-U_0)}{\gamma_0} \tag{3}$$

where U_0 is the normal potential and γ_0 is the normal gravity (Torge, 2001).

Absolute GPS height determination is based on the measurement of ellipsoidal heights, with subsequent reduction to given height system. It presumes application of a gravity field model which enables determination of geoid undulations in respect to a conventional reference datum. The undulations must be determined in grid of sufficient density covering the survey area, and must be of sufficient quality, to allow for retaining of an appropriate height accuracy level. Most of the practical GPS surveying applications are of extent of few kilometers, or of tens of kilometers. For such cases there is often used another procedure based on transformation with help of identical (common) points in both height systems. So as the vertical component of the GPS baseline is more sensitive to error sources and various disturbing influences, accuracy improvements are possible by optimization of measurement and evaluation procedures, and by detailed modelling of all the relevant influencing factors, resulting in elimination or mitigation of their effects. An example of measurement optimization may be the modified GPS observation procedure as proposed in (Švábenský and Karský, 1999) and further described in (Weigel and Švábenský, 1999), (Švábenský and Weigel, 2004).

2. MODIFIED GPS OBSERVATION PROCEDURE

It is commonly known that to achieve overall 1 cm accuracy level in vertical component, global GPS surveys working with baselines up to several hundreds of kilometers need several days sessions, regional surveys with baselines within range from 20 to 100 km need appropriate session intervals about 48 hours, while most of practical local GPS surveys of an extent below 20 km can hardly be shortened under 12 hours. Such long observing sessions are demanding on organization and costs. It was the main reason for proposition of a special measuring procedure which is based on the combination of data observed in shorter sessions, separated by constant time intervals, with minimized loss of accuracy (Švábenský and Karský, 1999). It is convenient to combine two (dyad), or better three (triplet) shorter sessions of 60-90 minutes duration, measured in separation of six to eight hours after each other.

Ideas of benefit from combination of shorter observation intervals took shape in course of the processing of long data series observed during the Sněžník network campaigns in 1998 and 1999. Local experimental geodynamic Sněžník network was established in 1992, in Czech-Polish cooperation. The network is situated along the Czech - Polish frontier in Králický Sněžník mountain region, in northern Moravia (for layout of the network see Fig. 4). Originally it was designed as testing local satellite network for monitoring of deformations in the upper layer of litosphere. Since 1992 yearly campaigns of GPS, electronic distance measurements, levelling, gravimetric, astronomical and other measurements had been carried out there within several research projects. Brno University of Technology had organized observation activities and carried out the measurements and processing of all campaigns in Czech part of the network since 1994, with help of standard Leica SKI-Pro and scientific Bernese GPS software. Among others also the problems of GPS heighting were investigated here, in cooperation of BUT and Research Institute of Geodesy, Topography and Cartography Zdiby - Geodetic Observatory Pecný (Švábenský and Karský, 1999), (Švábenský and Weigel, 2002).

Some investigations were carried out with aim to detect some regularities and interdependences in the coordinate time series. The autocorrelation function which describes the correlation of a variable with itself over successive time intervals (lags) was used in the following form (Box and Jenkins, 1976)

$$r_k = \frac{\sum_{i=1}^{n-k}(Y_i-\overline{Y})\,(Y_{i+k}-\overline{Y})}{\sum_{i=1}^{n}(Y_i-\overline{Y})^2} \tag{4}$$

where the observations at times X_i, $\overline{Y}$ is the mean value, and k is the time lag.

The example of autocorrelation of B, L, h series for baseline VYHL – BRAD (14 km) in Sněžník geodynamic network is shown in Fig. 1. It is clear from the graphs that the h component is behaving somewhat differently than the horizontal components

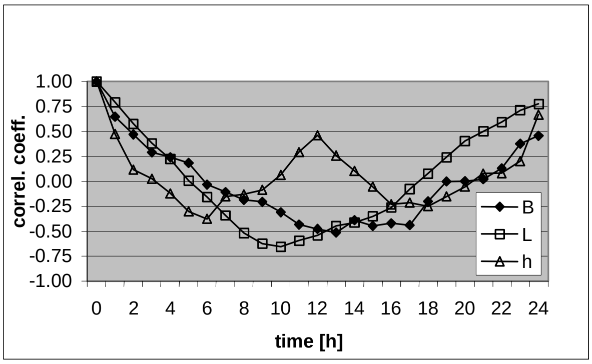

Fig. 1 Autocorrelation of B, L, h Series.

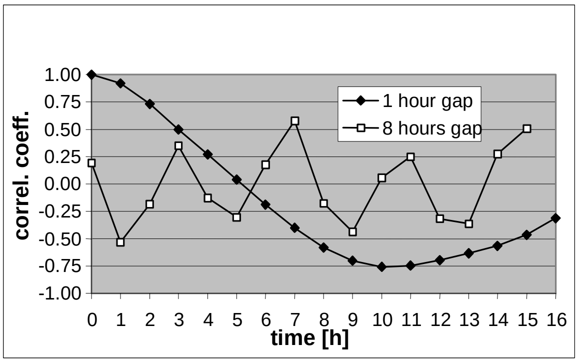

Fig. 2 Triplet Autocorrelations (L-Series).

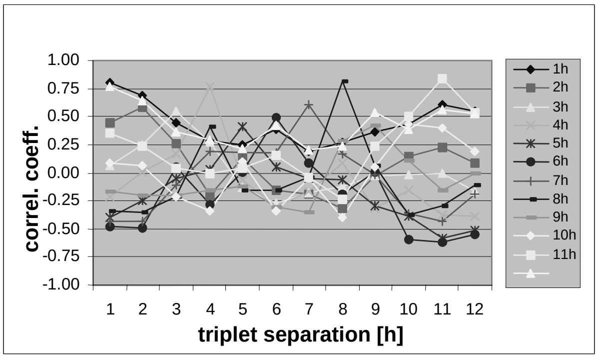

Fig. 3 Triplet Inter-Autocorrelations (h component).

B a L, and therefore closer attention was paid to autocorrelation studies. There were further investigated the autocorrelation functions generated for single and triplet series (which are naturally correlated). The example of comparison of the triplet autocorrelations for 1 hour and 8 hours separation intervals for L component of the same baseline VYHL – BRAD is shown in Fig. 2. The curve describing the 8 hours triplet time lag is oscilating around the zero value of the correlation coefficient in comparatively narrow zone, while the one after another following triplets with smaller or larger separation intervals are spreading into more wide range.

In Fig. 3 the comparison of the single autocorrelation functions graphs for different triplet separation intervals are displayed. Discernible is the narrowing of the graphs zones around the value of triplet separation interval of 6 hours. Similar narrowings for the horizontal components are around the 8 hours of separation interval. Comparisons of 24 hours results with triplet results can be found in (Švábenský and Weigel, 2004).

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3. PRACTICAL APPLICATION IN GEOIDAL MODELLING

The modified observing procedure for GPS heigh determination had been for the first time systematically used for the purpose of quasigeoid testing. Selected quasigeoidal section in direction of its maximal gradient in south-eastern part of Moravia was measured by GPS profile going along the state levelling lines of the first order. Processing of complete profile yielded mean standard deviation of a triplet height difference solution (computed from all segments) lower than 5 mm. (Kostelecký et al., 2002).

The GPS heighting procedure was further applied by measurements of two geoidal profiles (longitudinal in S – N direction, and transversal in E – W direction) in Sněžník network which follow the overall geomorphology characteristics of the upper Moravia river valley. Repeated measurements of the profiles were carried out by GPS and very precice levelling, in period 1998 – 2005. In Fig. 4 the profiles are marked by dotted lines, and the levelling lines measured within the BUT activities are marked by solid lines.

Main conditions for the data processing were 10 degrees elevation cut-off angle, ionosphere model computed from reference station data, a priori Saastamoinen troposphere model without estimation of corrections. So as the processed baselines did not exceed a few km in length, only the differential ionosphere and troposphere effects remained which could be kept at much more low levels that in cases of baseline lengths of ten and more km. Longer baselines need different approaches as to ionosphere and troposphere estimation, see e.g. (Borkowski et al., 2002).

Simultaneous employment of the GPS a VPL height measuring technologies made possible the creation of simple quasigeoid model, with its heights determined as differences between the GPS heights

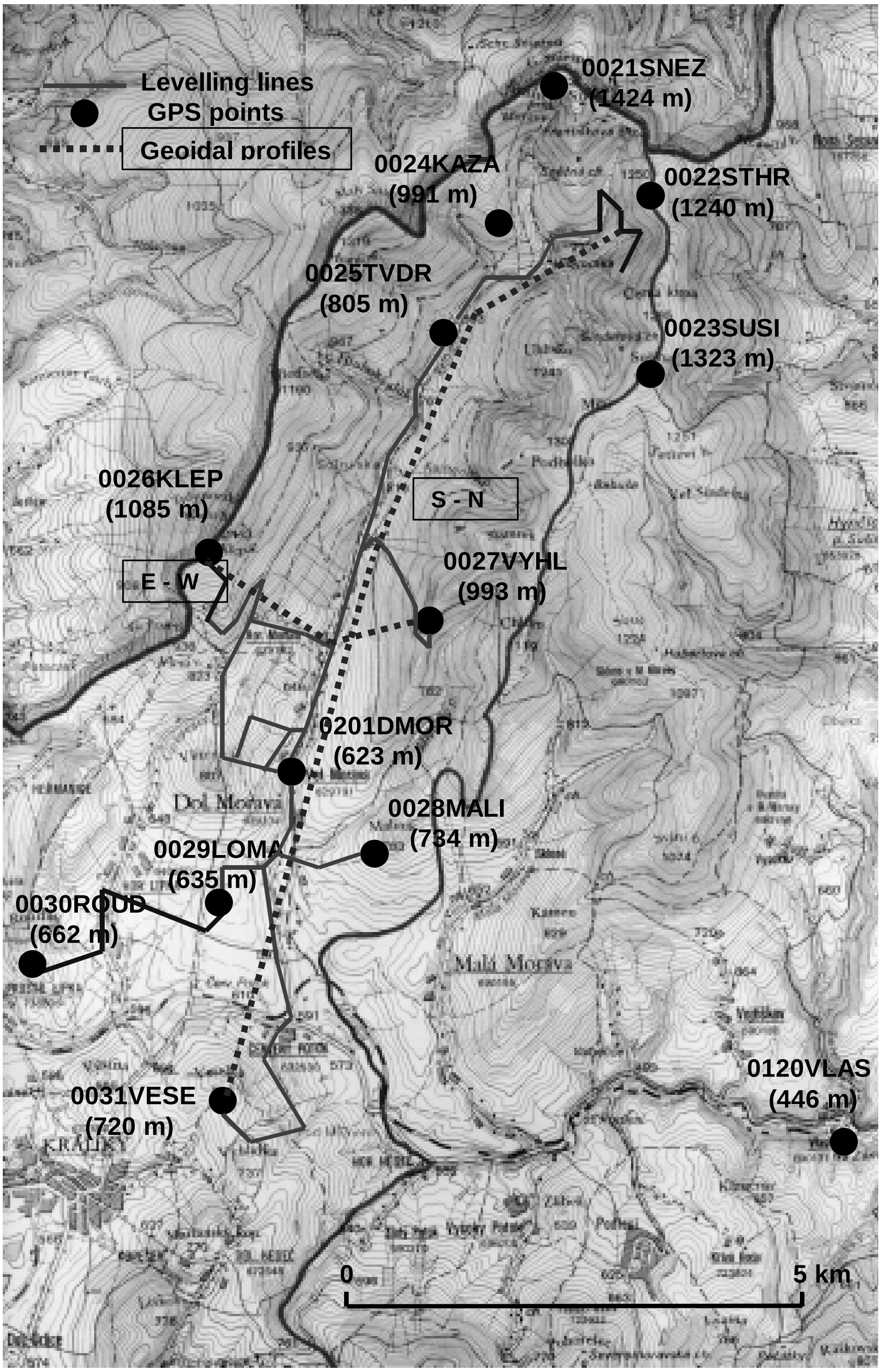

Fig. 4 Levelling Lines and Geoidal Profiles in Sněžník Network.

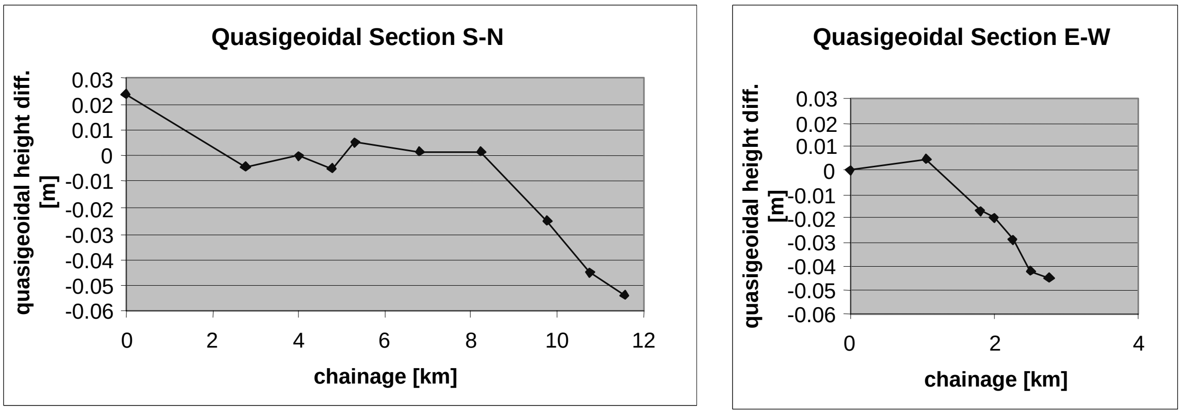

Fig. 5 Two Quasigeoidal Model Sections in Sněžník Network.

and the normal heights determined by VPL. The courses of the model sections along the both profiles measured are shown in Fig. 5.

4. CONCLUSIONS

GPS heighting is not quite simple procedure because it is combining satellite and terrestrial measuring methods together with gravity field determination (GPS, levelling, geoid modelling). Nevertheless, it becames a currently used technique in practice. When the accuracy has to be comparable with that of the lower order levelling, the observation times must be prolonged, in relation to baseline length and to other circumstances. The designed modified observing procedure for small scale GPS heighting based on combination of three shorter sessions with equal separation intervals of optimized length (triplet) offers an alternative to long and cumbersome static observations. The procedure is demanding as to organization and precise transport timing, but the final results indicate substantial productivity increase in GPS heighting. If single sessions of 1 – 1.5 hours duration are observed, it is possible to measure 4 - 5 stations in 24 hours using 2 receivers, and 15 – 20 stations using 5 receivers. In normal conditions the achieved accuracies (repeatabilities) of height differences are under 5 mm for baselines up to 5 km, and under 8 mm for baselines up to 15 km. On the other hand, the sessions used are comparatively short, which emphasize the importance of reliable ambiguity resolution. Meaningful is also the detection and mitigation of possible multipath or diffraction effects. Important for absolute heights determination with GPS is the exact knowledge of the local (quasi) geoid, especially in mountain areas with broken terrain relief.

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